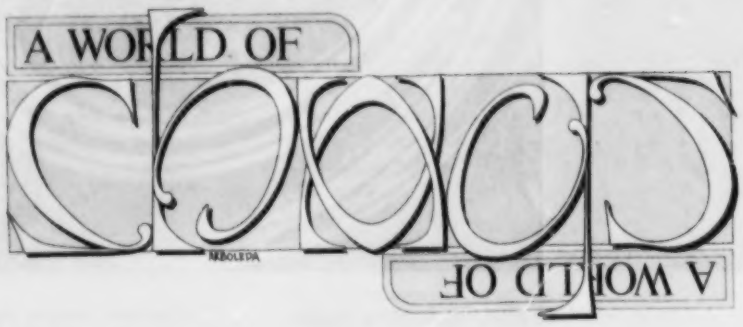
The United Nations University

Work in Progress

Volume 14, Number 1, June 1992



A World of CHAOS

Science traditionally has identified itself with order, regularity and a clear chain of cause and effect – a legacy owed perhaps above all to Sir Isaac Newton and his 17th century vision of a precisely determined cosmology. Even Albert Einstein, in his pursuit of relativity in the early years of the present century, could not abandon the vision: while others might see a universe where God played dice, he believed in one governed by law and order.

In the past several decades, however, a new challenge has arisen to Newtonian determinism. Scientists in a number of fields are coming to understand that things may not be nearly so neat and tidy as they once assumed. Increasingly, the systems which govern life are being recognized as characterized by complex instabilities, fuzziness and disorder. Our world, it appears, is a place of considerable chaos.

The term "chaos" is used by scientists not in its conventional sense of total bedlam, but rather in a specialized way. Chaos theory deals essentially with the concept of order within disorder. Turbulent, seemingly chaotic systems – governing, for example, the swirl of a fluid or the waft of a smoke ring – may look as if they behave randomly. But, in reality, the new thinking suggests, they exhibit an underlying long-term order that is neither wholly deterministic nor wholly random.

As the British mathematician Ian Percival notes, the budding school of chaos is "a river that has been fed by many streams." Involving a broad array of academic disciplines, it emphasizes the importance of an interdisciplinary approach when probing the frontiers of knowledge. This is one of the reasons the new field attracted the interest of the United Nations University. Two other notions undergird the thinking about chaos, both of which are intimately interlinked with the UNU approach to problem-solving:

- small changes can lead ultimately to larger, often unforeseen changes. This "signature of chaos" might well also characterize today's interdependent world, where a minor action or decision in one corner of the globe can quickly translate into major impacts elsewhere;
- much of reality is a condition of persistent instability and change, which is as much a prescription for most of today's global society as it is for patterns of chaos.

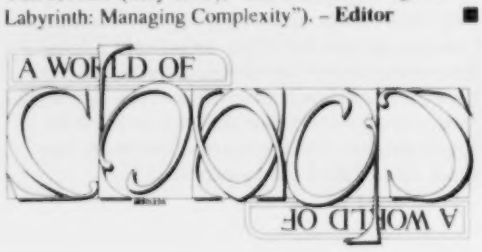
The UNU organized an international symposium on the Impact of Chaos on Science and Society that was held in April 1991 at the University of Tokyo, the first of a series of seminars on the frontiers of science and technology. Under the coordination of Prof. Celso Grebogi of the University of Maryland, USA, the UNU brought together many of the seminal thinkers in this burgeoning field. This issue of WORK IN PROGRESS presents excerpts from papers presented at the symposium.

While it is positioned out on the cutting edge of knowledge, chaos theory has a lot of potentially hard-nosed applications. It has, for example, potential relevance to our understanding of weather forecasts, crop cycles, heart rhythms, market fluctuations, earthquake predictions, ship behaviour in heavy seas, and the spread of the AIDS epidemic.

The main four selections in this issue are by leading trailblazers in chaos thinking. David Ruelle, who made notable contributions to the exploration of chaos in mathematical physics, discusses how the interplay of these two disciplines led to new insights on chaos. Mitchell Feigenbaum, one of the field's most prolific thinkers, describes the process which brought him to recognize the hidden unity in chaotic behaviours. Yoshisuke Ueda relates how, in 1961, he may have been the first to see, in his Kyoto laboratory, the bizarre and beautiful image later called a "strange attractor." Hao Bai-lin, who compiled many of the historical papers on chaos into a single reference volume, discusses the various scientific roots of chaos theory.

The remaining selections take up some of the specific areas in which chaos theory might be applicable. David Campbell and Gottfried Mayer-Kress consider its implications for helping to cope with the unpredictabilities of the arms race. Michele Boldrin examines how it may provide new insights about economic fluctuations. V.I. Keilis-Borok notes some of the linkages between deterministic chaos and nature's signs of an impending earthquake. Steven Bishop discusses connections between engineering dynamics and chaotic behaviour which could help explain why some ships unexpectedly capsize. Renate Mayntz reviews the implications of chaos theory for the sociologist and the ways in which societies order themselves. Finally, there are two accounts of the tantalizing relationships there may be between chaos and physical and mental health: Leon Glass discusses his experiments on the problem of irregular heart beats. Arnold Mandell and Karen Selz consider the trail of chaos in brain waves.

The UNU's interest in chaos phenomena stems from its concern, in its Second Medium-Term Perspective, with the impact that new initiatives in science and technology could have on the social and natural environments. Chaos theory specifically is one of the tools in studies of complexity, an area in which the University has already carried out extensive research (see the volume, *The Science and Praxis of Complexity*, published by the UNU in 1985, and *UNU Newsletter* Vol. 8, No. 3 (May 1985), "The Threads through the Labyrinth: Managing Complexity"). – **Editor** ■



UNU's New Logo – Part of Its Recently Launched New Visual Identity

Variations of the Logo Will Identify the University's Research and Training Centres and Programmes

During 1991 the United Nations University invited Banks & Miles, a London based design practice with an international reputation and considerable experience of working with academic institutions, to look at the way it was presenting itself to the outside world and develop a new visual identity for the UNU. The new visual identity is intended to be something that can be used throughout the University to reflect the quality and aspirations of the whole institution to a wide range of publics in many different countries. A strong and consistent visual identity is one of the tools available to an organization to help it to be clearly recognized.

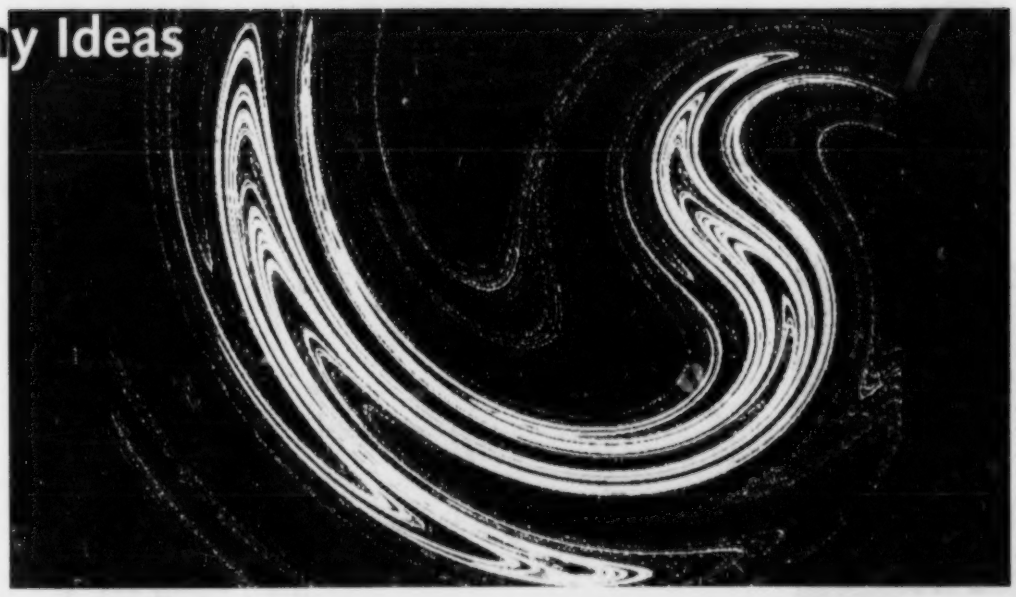
The United Nations University Headquarters in Tokyo and its research and training centres and programmes (RTC/Ps) and offices in different countries have been using different formats, symbols and colours and a variety of lettering styles. As a result, the documents and publications emanating from the University do not present a coherent image. So the reputation gained by the successes and achievements in one area did not fully reflect credit on the University as a whole.

Finding a symbol for the UNU was not easy. Being both an academic institution and an organ of the United Nations with its headquarters in Tokyo, and research and training centres located worldwide, it was necessary to reflect this diversity. However, it was equally important to project an image of a distinctive, accomplished, unified and modern organization confidently working towards clear objectives. What was needed was a visual trigger which could quickly become associated with the University. Origins of the symbol lie in the letters "UNU" and the idea of a worldwide academic network. The main features of the overall design are the symbols of the UN and the UNU and the name style of the United Nations University.

The new visual identity will be introduced in mid 1992 in preparation for the UNU's move to its new headquarters building in Tokyo. The need to produce new stationery, publicity materials and signs as a result of this move has enabled the UNU to introduce its new visual identity in a timely manner and at minimal cost.

Work in Progress of the United Nations University
53-70, Jingumae 5-chome, Shibuya-ku
Tokyo 150, Japan
Telephone (03) 3499 2811 Fax (03) 3499 2828
UNU TOKYO

The Interplay of Many Ideas



"Chaotic attractor for the Ikeda map." Photograph by Celso Grebogi

By David Pierre Ruelle

In the early 1970s, scientists began to take a look at disorder to seek connections between different kinds of irregularities. One of this small, scattered band of researchers was David Ruelle, a Belgian mathematician working at the Institut des hautes études scientifiques in Paris. His concern was with the onset of turbulence: could a scenario be plotted which would help explain a sudden wild swirl of smoke or a seemingly unexplained change from steady dripping to disorderly flow in a leaking faucet?

In 1971, Ruelle and a visiting Dutch mathematician at his institute, Floris Takens, wrote a paper, "On the Nature of Turbulence," in which they argued that just three independent motions were sufficient to produce the full complexity of turbulence. The three motions, or frequencies, combined to create a new object – which Ruelle and Takens called a "strange attractor." In the paper he presented at the Tokyo symposium, Dr. Ruelle discussed the interplay of ideas coming from both physics and mathematics which has led to remarkable new insights about the nature of chaos. – **Editor**

One can hardly discuss the impact of chaos on mathematics and ignore its reverse – the impact of mathematics on chaos. Looking at the problem more carefully, one realizes that there is indeed a rich history of interactions between the mathematical theory of dynamic systems, and its applications to the field of chaos. Chaos, we can see, is basically a mathematical phenomenon.

As we have come to know more about the condition or state we define as chaos, it seems clear that one of the most important factors that must be present is sensitivity to initial condition. Put another way, the prevalence of sensitivity is what we now call chaos. Chaos, thus defined, was discovered by J. Hadamard at the end of the 19th century in his studies of the shortest distances between two points on surfaces of negative curvature. Hadamard immediately understood the philosophical importance of his result: an arbitrarily small uncertainty in the initial condition entails a large uncertainty in the predicted state of the system after a sufficiently long time.

Other scientists, among them Henri Poincaré, also understood the importance of the phenomenon discovered. Poincaré discusses the relevance of sensitive dependence on initial conditions to the dynamics of three or more spheres in space system as well as to weather predictions.

The early discovery of chaos, however, had no lasting influence on physics. The new ideas were forgotten and had to be rediscovered again, much later and independently. On the mathematical side, however, the work of Hadamard and Poincaré led to uninterrupted progress up to the present day with the contributions of people like Andrei Kolmogorov in the field the kinetic theory, Stephen Smale in topology, and many others. An essential step in the mathematical development of dynamical systems theory was the creation of ergodic theory – the concept that any system will arbitrarily return very close to its original state over a sufficient period of time.

New Tools Available

The time evolution of chaotic systems is typically complicated and irregular-looking. Indeed, a regular time evolution is predictable and therefore not chaotic. When the interest in complicated and irregular time evolutions arose among physicists in the 1970s – leading to what is now called the theory of chaos – all kinds of new scientific tools existed that had not been available to Poincaré.

One such tool is the electronic computer, which allowed the meteorologist Edward Lorenz to compute at the Massachusetts Institute of Technology in 1963 a chaotic time evolution and visualize it in the form we now call a "strange attractor." Other tools, like ergodic theory, were mathematical. Finally there were new experimental tools permitting, for instance, a detailed study of the onset of hydrodynamic turbulence. It is this experimental study which showed that hydrodynamic turbulence is chaotic that corresponds to the strange attractor phenomenon described in the paper which Floris Takens and I published in 1971.

It must be said at this point that, however insightful and brilliant they were, the physical ideas of Poincaré on chaos were at the level of scientific philosophy. Because of the new tools now available, our present understanding of chaos in physics is at the level of quantitative science. And attempted applications of chaos to biology, economics, etc. – which are now at the level of scientific philosophy – may later reach the level of quantitative science.

I have stressed the idea that the study of chaos consisted in applying available mathematics to the understanding of natural phenomena. But there have also been important influences in the opposite direction: from the physics of chaos to mathematics. New mathematical tools were created because they were needed to understand chaos, or new ideas coming from chaos were found to have surprisingly important mathematical content, and were then developed for their own sake. In fact, the limit between physical and mathematical ideas is often unclear, and the same people have been seen sometimes to function in physics, sometimes in mathematics.

Let me now list some mathematical developments where the ideas of chaos played a significant role. The list does not attempt at being exhaustive, and the scope and nature of the items is rather different.

The Lorenz Attractor

The mathematical studies of the American topologist Stephen Smale have shown that the orbit of a dynamical system is in some cases asymptotic (that is, a curve approaching but never touching a straight line) to a complicated set called an Axiom A attractor. The behaviour of the system is then chaotic and, since Axiom A attractors can be analysed in great details, they provide very important examples of chaos.

Another example of chaos was obtained in an early computer study by Edward Lorenz when he analysed a rather brutally simplified model of convection, the circulation of fluids and accompanying changes in heat and density. The Lorenz attractor is chaotic, but different from the Axiom A attractors of Smale in that it contains an unstable fixed point for the time evolution. Interestingly, at this time we have no mathematical proof that the solutions of the Lorenz system behave chaotically as we think they do. We have, however, a model inspired by the equations, which is called *geometric Lorenz attractor*. It is known in particular that the geometric Lorenz attractor has some properties of persistence when the equations are slightly changed.

The Feigenbaum Bifurcation

When a dynamical system depends on a parameter such as those in Lorenz's equations, the qualitative behaviour of the system may change when it goes through certain special values. Such a change in qualitative behaviour is called a *bifurcation*. Apart from their mathematical interest, bifurcations play an important role in applications. An example is the Hopf bifurcation which replaces an attracting fixed point by an attracting periodic orbit. This results in the replacement of a steady state of the dynamical system by a periodic state. Eberhard Hopf, who studied this bifurcation in detail after the early work of Poincaré and Aleksandr Andronov, was aware of its relevance to hydrodynamics.

It has been noted by various people that one can sometimes observe period-doubling cascades – as from period T to 2T, 4T, 8T, etc.... The period doublings also accumulate, to a limit, as the mean is varied, and they do so in geometric progression. The great merit of the work of Mitchel Feigenbaum has been his understanding of the principle for this cascade of period doublings in geometric progression. The various period doublings are scaled down versions of the same phenomenon. A new technique, imported from the physics of phase transition, is that of *renormalization*. Its concepts give the key to the period-doubling cascade. (Central to the theory of renormalization is the notion of self-similarity – that when a small part of a whole is picked out and greatly enlarged, the whole will be very closely approximated, or "renormalized." Thus properties of the original whole can be determined from the finite geometry of the "renormalized" subject. – **Editor**)

Hydrodynamic Turbulence

The phenomenon called turbulence is complex and difficult to analyse. What we know about it results from experimental, numerical and mathematical studies. One important feature of turbulence, as Takens and I have tried to demonstrate, is that it is chaotic. But *this has been proved only experimentally* by the observations of strange attractors, Feigenbaum bifurcations, etc. at the onset of turbulence. A mathematical proof of chaos in the Navier-Stokes equation (for the deterministic flow of viscous fluid) does not exist at this time – and when one is obtained, it will no longer create much excitement. Thus we have here a very interesting situation from the epistemological viewpoint, where we are firmly convinced of a certain mathematical fact (the existence of chaos in the solutions of the Navier-Stokes equation), but our belief is based on experimental evidence.

When Chaos Meets Math

These have been only a few examples of encounters between chaos and mathematics. Many others could be cited – in, for example, the area of ergodic theory or quadratic mapping or dynamical zeta functions. These are all rich and complex areas of research. I hope the areas I have discussed show how varied and diverse this field is – showing that the relations between mathematics and the other sciences can take all kinds of different forms.

To talk about the impact of chaos on mathematics is to suggest a one-way relation, with mathematics at the receiving end. A card-carrying mathematician could well argue in the opposite direction, and quote the party slogan about mathematics being the "queen and servant" of other sciences. What I have tried to show is that, in the case at hand, the relations of mathematics with physics (or more generally the natural sciences) have been more subtle and interesting than one might a priori imagine. Physics has provided some questions *and* some answers to mathematics. Mathematics has provided its own way of thinking about problems. Physicists have been functioning as mathematicians and vice versa – and the result has been a decade or two of exceptionally fruitful intellectual work, with benefit both to mathematics and to the physics of chaos. ■

Chaotic Movement in the Economy

By Michele Boldrin

Economists have traditionally taken the view that, other things being equal, the market should consistently course. Its fluctuations, by this reasoning, are due primarily to the impact of outside forces – new technologies, shifting consumer tastes, the vagaries of government policy, and so on. In the late 1970s, however, certain economists began to explore the possibility that persistent oscillations and chaos might be inherent in dynamic economic models.

A major achievement of this research has been to show that such fluctuations would not be incompatible with economic rationality. Some of the work being done in this as yet highly theoretical area was discussed at the symposium by Michele Boldrin of the Department of Managerial Economics and Decision Sciences, Kellogg Graduate School of Management, Northwestern University, Evanston, Illinois, USA. – **Editor**

Recent work has seen a revival of interest in the hypothesis that aggregate fluctuations and other exotic economic phenomena might have inherent characteristics that would persist in the absence of outside random "shocks" to the economy. Even without giving credence to the extreme (and surely implausible) view that macroeconomic functions can be interpreted as purely deterministic, the possibility that exogenous shocks might play a relatively minor role in generating market movements must be judged of no small importance.

This hypothesis of cycles being caused from within the system is not totally new. Indeed, the earliest formal models of business cycles were largely of this type. In these early models, the stationary growth path of the economy is locally unstable, but deviations from it are eventually contained by "floors and ceilings" – such as shortages of productive factors on the upside or technological limits to the amount by which investments can be made negative on the downside.

By the late 1950s, however, this way of attempting to model total fluctuations had largely fallen out of favour. The dominate approach, instead, was that which saw exogenous, stochastic or random "impulses" transformed – via the economy's "propaganda mechanism" – into a characteristic pattern of oscillation.

A major reason for the decline from favour of the endogenous cycle hypothesis concerns the inadequate behavioural foundations of the early models of this kind. The stability results obtained for many simple equilibrium models ... led many economists to suppose that the models were not only lacking explicit foundations, in terms of optimizing behaviour, but also depended upon assumptions that were necessarily inconsistent with optimization.

Dismantling this long-held prejudice has probably been the single most important achievement of the recent wave of studies in nonlinear economic dynamics. What the last 10 to 15 years of theoretical research has done is to pick up the standard, commonly-accepted models of periodic equilibrium and show that chaotic fluctuations were perfectly consistent with them. The role of bifurcation theory in demonstrating how and when periodicity gives way to chaos was essential to this research effort. Economists have learned to take seriously the functional forms they use in their models, give them appropriate parameters, and then find that, as the parameter values moved, new and unexpected configurations of the phase space would emerge.

No area of economic theory and no class of models has been exempted by this revisionist enterprise. It now seems widely accepted, among economic theorists at least, that chaotic dynamic patterns may theoretically arise in a well-functioning market economy.

The progress made, nevertheless, stops exactly here. We have still not shown that chaos does emerge in accepted economic models at empirically relevant parameter values and – more importantly – that the time series so obtained "match" (in any reasonable sense) the measured statistics of actual economics.

Indeed, it appears that, from this point of view, we are still in our infancy. Very little has been done to show that chaos may be a useful tool to interpret real economic phenomena. In particular, the "empiricist" approach to the problem – which has tried to provide certain dimensions to various raw economic data – has not yielded the positive answers that were initially expected. This is not to say that this line of effort has been useless and that no further progress can be made. For one thing, our efforts to date have made the statistical foundations more mathematically rigorous.

Learning from Models

In our analysis of a simple general equilibrium model of the economy, we have been able to generate both cyclical and chaotic paths for variables such as output, consumption and investment. We characterized the dynamic behaviour of our economy in terms of its basic parameters: (a) the labour share of income in the consumption sector; (b) the capital/labour ratio in the investment sector; and (c) the discount factor.

For many of our parameter values, a more or less steady state obtained. Successive bifurcations – abrupt changes in behaviour – then led to a chaotic regime, but only for rather unrealistic values of the parameters. Since our model was testable, we ran some simulations for the chaotic regimes, and compared their statistical properties to those of the postwar US economy.

At a qualitative level, the simulated time series behaved fairly well, but quantitatively left something to be desired. This should not be too surprising for as we have discovered, the analytical complexity of nonlinear models greatly exceeds that associated with linear ones. The primitive stage of our research technology forced us to work with a rather rudimentary and rigid model. The introduction of an elastic labour supply, a nonlinear utility function, and increasing returns to scale in production are all elements of reality that could improve the performance of our model. Future research on models with a higher dimensional state space may also be more successful, since it appears that even a slight departure from linearity may produce strange attractors.

While still in its infancy, the study of nonlinearities in economic models is likely to provide insights into the forces behind observed economic fluctuations. In our model, we underlined the importance of the effects of substitutions between the sectors (introduced by different degrees of profitability). We also considered the impact of different time periods in determining allocation decisions, investment activities and so on. Bypassing the nonlinearities with first order approximations would have neglected the important contribution of factors such as these in amplifying and sustaining oscillations in the economy. ■

From the Floor: Chaos and the Space Telescope

The contribution that chaos theory may have made to the successful launch of the US space telescope in 1990 was cited in one of the panel discussions by Dr. Celso Grebogi of the Institute for Physical Science and Technology at the University of Maryland (USA); he was commenting on the interactions between different disciplines in the development of chaos theory. Dr. Grebogi and his institute colleague Dr. James Yorke helped organize the Tokyo chaos meeting and are co-editors of the UNU volume of proceedings which will be published. Dr. Yorke is the person credited with giving chaos theory its name, in an article he published in the *American Mathematical Monthly*, "Period Three Implies Chaos." The term came to stand for the whole field of deterministic disorder. – Editor

Grebogi: The initial impact of chaos was in the more exact sciences – and I think we have seen here that chaos was perhaps better understood in those areas. But we also see how chaos is permeating the less exact sciences, like the social sciences, economics, etc. This symposium, I believe, truly attests that chaos is interdisciplinary – with mathematics perhaps at the helm.

As Jack Wisdom emphasized over and over, chaos has become crucial in understanding many lesser known phenomena in both the sciences and engineering. Let me mention one example of a practical application – and that is in the launch, in April 1990, of the US space telescope. One of the problems was in determining the intensity of the solar cycle which, in effect, protects the atmosphere, during which the launch should take place. The best prediction was one using David Ruelle's techniques. This predicted the largest and strongest cycle in this century – during which NASA tried to put the space telescope as high as possible. This is perhaps a contribution worth citing – worth maybe 200 million dollars – since NASA will not have to go back again and boost the height of the space telescope. I think that this shows there has been cross-fertilization between engineering and mathematics – and both are benefitting from it.

TITLES OF INTEREST

The Impact of Chaos on Science and Society

Edited by James Yorke and Celso Grebogi

This volume looks at some of the repercussions which the concept of chaos has had on a wide variety of scientific disciplines and how this new perspective has changed our understanding of natural and social environments.

To be published

The Science and Praxis of Complexity

By Ilya Prigogine, S. Aida, J.-P. Dupuy, K. Pribram, J.-L. Le Moigne and others

This volume discusses recent findings and approaches in different disciplines and areas of research of intellectual development concerned with the nature and behaviour of complex dynamic systems and the phenomena of complexity.

ISBN 92-808-0560-6 GLDB-2/UNUP-560

1985 US$25; developing country price: US$12.50

UNITED NATIONS UNIVERSITY PRESS

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The Origin of Chaos

By Yoshisuke Ueda

In structurally stable systems, mathematicians identify two typical "attractors" – one representing behaviour that reaches a steady state or fixed point (as, for example, the spot where a free-swinging pendulum eventually comes to rest), the other a cycle that repeats itself continuously. Some of the first footprints of chaos appeared to researchers in what came to be known as "strange attractors."

One of the first to focus on the problem of the strange attractor was Professor Yoshisuke Ueda of Kyoto University. In 1961, as a graduate student in electrical engineering at Kyoto, he encountered the strikingly beautiful image of a strange attractor while doing computer simulations of an oscillator. In the paper he presented to the April 1991 chaos symposium, Dr. Ueda discussed his own personal experience in helping to identify the role of strange attractors in the origin of chaos theory. – **Editor**

People say that the data I was collecting with my analog computer on the 27th of November 1961 is the oldest example of chaos discovered in a second-order non-autonomous periodic system. Around the same time it was Lorenz who made the discovery of chaos in a third-order autonomous system.*

At the time, I was simply frustrated with this seemingly mysterious phenomenon which I accidentally came upon during my experiments. For my part, it was nothing as glorious as an act of discovery. All I did, for a long period of time, was to keep on pursuing my stubborn desire to understand this unsettling phenomenon. What are the possible steady states of a nonlinear system (that is to say, one where relationships are not strictly proportional)? And my paradigm has always been the phenomena themselves – not papers with their abstractions, but something we can actually observe or quantify.

As I remember that November day in 1961, I had just finished writing the narrative to accompany the data I was going to publish at the Special Committee on Nonlinear Theory of the Institute of Electrical Communication Engineers, to be held in December, and was carrying out some analog computer experiments in order to test the applicability of the approximate computation I quoted in my paper. At that time, I was a third-year graduate student at Kyoto University, working on the phenomenon of frequency entrainment under the guidance of Professor Chihiro Hayashi. Frequency entrainment is a phenomenon that occurs when a self-oscillating frequency is attracted to and synchronized with that of an external driving frequency.

The main purpose of my computer experiment was to simulate the nonlinear differential equation describing frequency entrainment. I also sought to examine the range of the frequency and amplitude of the driving external signals which resulted in synchronization, as well as the amplitude and the phase of its oscillation. (For the non-scientist, a linear equation is one in which the sum of two solutions provides another solution; nature, however is virtually all nonlinear, and the behaviour of linear equations is far from typical.) I made an approximate computation by rewriting the non-autonomous equation into an autonomous one using the averaging method – which had the result of suppressing any evidence of chaos.

As we now know, there are actually two kinds of asynchronous oscillations – quasi-periodic (represented by a limit cycle attractor in the averaged equation) and chaotic oscillation (represented by strange or chaotic attractors). In those days, however, only the equilibrium point and limit cycle were known to exist as steady states, or attractors, of an autonomous system. So it was understandable to have the preconception that an asynchronous condition meant quasi-periodicity.

The Broken Egg

On that November day when I changed the parameter – the frequency of the driving input – and the condition shifted from frequency entrainment to asynchronization, the oscillation phenomena portrayed by my analog computer was chaotic indeed. It was nothing like the smooth oval closed curves I had been seeing earlier, but was more like a broken egg with jagged edges. My first concern was that my computer had gone bad. But I soon realized that this was not the case. It did not take long for me to recognize the mystery of it all – the fact that during the asynchronous phase, the shattered egg appeared more frequently than the smooth closed curves, and that the order of the dots which drew the shattered egg was totally irregular and seemingly inexplicable.

One day in the fall or winter of 1962, Professor Hayashi asked me to do some computation involving third order simultaneous equations with four unknowns to draw amplitude curves of the Duffing equation. It was particularly difficult on the hand-cranked calculator, but I was somehow trusted by Professor Hayashi as someone having fast and accurate computation and careful drawing skills. As it was impossible to solve these equations with four unknowns by sheer brute force , I eliminated two variables, gave the amplitude ahead of time with a bit of "manipulation," and finally, by hand calculations, solved approximately 50 cases of third order equations. This crash project was completed within a few weeks. The damping coefficient was set at 0.2.

The amplitude characteristic curve drawn with only two frequency components was, of course, nothing but an approximation. The standard procedure, therefore, was to verify the results with the analog computer. During this process, I struggled with enough chaotic oscillations (the source of the Japanese attractor) to make me sick. But Professor Hayashi told me: "Oh, it's probably taking time to settle down to the subharmonic oscillations. Even in an actual series resonance circuit, such a transient state lingers for a long time." When I look back though, I seemed to have sensed at that time that chaos was not a phenomenon unique to forced self-oscillatory systems in which quasi-periodic oscillation appeared.

Only a fragmented record of the original data of my work with the analog computer and manual calculation remains now. I may have been worried at that time that, had Professor Hayashi seen this data, he would have told me to repeat the analog computer experiments until the transient state settled to a more acceptable result. Sensing that no matter how long I continued the simulation, I would never be able to come up with the data he wanted, short of making up some false data, I must have suggested using a larger damping coefficient of 0. 4 (Chaos does not occur for this large coefficient.) and done away with the problematical data.

I finished my doctorate in the spring of 1964 and was hired as a research assistant in the Department of Electrical Engineering at Kyoto University. During that year, Professor Hayashi's book, *Nonlinear Oscillations in Physical Systems*, was published by McGraw-Hill. The Hayashi Laboratory was overflowing with chaotic data produced by the analog simulations, and yet they were overlooked and thought to be quasi-periodic oscillations or transient states.

Enter the Digital Computer

The digital computer KDC-I became available to us in the laboratory at about that time. I used it to compare results, such as the periodic solution of the Duffing equation, with the analog data. The KDC-I was a machine built with transistors, and took about 60 seconds to integrate the Duffing equation along the time axis from $t = 0$ to 2π, using the Runge-Kutta-Gill method that set the size of integration step at $2\pi/60$. It wouldn't even make a toy today, but at the time I was deeply impressed by the fact that such a calculation – impossible to do by hand – was finally possible.

In our laboratory, mapping or the stroboscopic method had been used for analog computer experiments. In modern terms, hand-made automatic mapping is a device which uses the analog computer to create Poincaré maps.* (These maps, in effect, take a slice from the heart of the attractor, removing a two-dimensional section in much the same way as a pathologist prepares a section of tissue for the microscope. It enables the investigator to sample motion at given intervals – for example, the velocity of a pendulum bob each time it passes through its lowest point.) The device plots on recording paper a sampling of the analog signal at the same instant during each cycle of external force. Thanks to the automatic mapping device, drawing invariant curves became much easier. As a result, the application of discrete (stroboscopic) dynamical systems theory to nonlinear oscillation was speeded up considerably.

During the latter half of the 1960s, work continued at the Hayashi Laboratory on nonlinear oscillations. Most of the research data from our group were published by Professor Hayashi at international meetings. Then during the late 1960s, I sent a paper of my own for the first time to the *Journal of the Institute of Electronics and Communication Engineers*. (This was a time when the university was in turmoil with student protest; the democratic atmosphere that prevailed on campus might have prompted me to take this action.)

Concept of Chaos Established

In the paper, I was able to demonstrate quasi-periodic oscillation and chaos in Rayleigh-van der Pol mixed type equation. Though it hadn't taken clear shape yet, the concept of chaos was already established in my mind at that time. This fact can be supported by an oral report made at a seminar on "Ordinary differential equations and nonlinear dynamics," held in December 1970 at the Research Institute for Mathematical Sciences at Kyoto University, under the leadership of Professor Minoru Urabe.

The record of the comments I made at the end of my research report still remains: "... However, according to my observation of the phenomenon with the use of a computer, each of the minimal sets which make up the set of central points are all unstable, and the steady state seems to move randomly around the vicinity of the minimal sets, influenced either by small fluctuations in the oscillatory system or by external disturbances." These minimal sets are the unstable periodic motions in the attractor. The descriptions led to my proposed name, "randomly transitional phenomena," for the characteristics of chaos in a physical system.

I made the report in the Urabe seminar in the hope that the mathematicians might possibly support my interpretation of the random oscillations. But my gamble backfired, and Professor Urabe admonished me personally. "What you saw was simply the essence of quasi-periodic oscillations," he said. "You are too young to make conceptual observations." The existence of random oscillations – or chaos – was so obvious in my mind that this negative comment did not crush me. I was, of course, deeply disappointed that no one understood it no matter how hard I tried to explain – and from then on, I became even more careful in my research efforts.

Grasping the Concept

I sent off a second paper, based on my oral report, but it was rejected. I spent nearly a year rewriting it and sent it again in September 1972. It was finally accepted. The summary of the discrete dynamical system theory I included in the appendix was by far the most difficult work of the paper. I will never forget how nervous I was, wondering whether or not I had a full and accurate grasp of the concept.

The mathematicians who valued rigorous proofs were, in a way, my bane. They could set up any unrealistic assumptions in their heads and live in their world of abstractions, but we are living in the real world. While I wanted them to hear me out a little more sympathetically, I also idolized mathematics. ■

* Edward Lorenz, MIT meteorologist, who in 1961 made his computer runs on weather patterns which played a seminal role in the development of chaos theory.

* Named so after mathematician Henri Poincaré.

Chaos and Social Order

By Renate Mayntz

The neat and tidy determinism of Newton's view of the world has never really accorded with the global society's day-to-day realities – and a glance at the unrest and tumult reflected in today's headlines make that starkly apparent. The leading thinkers in the social sciences have all rejected determinism in one way or another, as they sought to explain how society organizes itself and copes with truth.

The issue essentially was how society moves from a state of disorder and confusion to one of higher order and rationality. The role that modern chaos theory might have to play in this process – and what its inherent limits are – was discussed at the Tokyo symposium by Dr. Renate Mayntz of the Max-Planck-Institute for Social Research in Cologne, Germany. – **Editor**

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Over the past few decades, attention in the natural sciences has increasingly turned to phenomena which defy analysis in terms of the traditional world view, with its assumptions of linearity and reversibility. More and more, the concern has been with the behaviour of systems remote from equilibrium and with discontinuous processes resulting from nonlinearity.

Nonlinear systems display a number of behaviours which can be widely observed in the natural world. Their variables can change discontinuously, producing so-called "phase jumps" – for example, sudden changes of state as in the phenomenon of ferromagnetism or in superconductivity. In discontinuous processes, threshold or critical mass phenomena often play a decisive role. Such a circumstance exists where a dependent variable reacts very little or not at all to changes up to a certain point – beyond which it reacts suddenly and strongly.

Human societies display many of the characteristic features of nonlinear non-equilibrium systems: unpredictability, complex interdependencies, delays in response, transitions from one state to another and the importance of a critical mass in producing and sustaining change. Social theory has always started from the assumption that disorder is a natural state.

Sociologists have been primarily concerned with the emergence of social order from this basic formlessness. This was true of social philosophy well before the advent of sociology proper. Thus Thomas Hobbes explained how the state was created as a means to curb the chaotic war of all against all which characterizes the natural state. Emile Durkheim found the basis of social order in the conscience of the collective, while Max Weber pointed to traditions, values and interests as the factors producing social regularities.

The rapid and radical losses of a state of order can be found at every level of social reality – from marriage disruptions to the failure of businesses to the destruction of political regimes. A sudden collapse of the social order is followed by a period of turbulence, where future developments can be largely subject to accidental events. This insecurity of social order has meant that Newtonian mechanism has never really been an accepted model for the representation of social reality. Sociologists thus often focus on situations where a state of order suddenly breaks down. The very discipline of sociology, in fact, is rooted historically in the French Revolution.

Over the years, a number of theoretical developments in the natural sciences have increased the potential relevance of physical theories for sociology. Particularly common were attempts to picture society as an organism – such analogies, for instance, inspired Herbert Spencer in his 19th century analysis of social differentiation. From there, an unbroken line of sociological theorizing about the structure and development of social systems can be traced to the very present.

Catastrophe Theory

Recent efforts have sought to model discontinuous social processes with the help of mathematical catastrophe theory. This was meant to provide an instrument for dealing with situations where the continuous variation of one or more parameters leads to sudden changes in the outcome.

Obviously, the sociological application of catastrophe theory must have been very tempting – particularly in view of the observation that seemingly small disturbances will often cause a social or an economic system to enter into a state of instantaneous, radical change – or even collapse. However, it is obvious that a catastrophe in the mathematical sense bears only a superficial resemblance to real life – where catastrophe usually takes the form of a system breakdown with severe consequences for the people directly or indirectly affected.

Catastrophe theory can be applied, however, to situations where relationships can be given strict empirical meaning. John Casti, for example, has succeeded in developing a formal model of changes in the growth rate of housing units. He shows mathematically that with a specific combination of parameter values, the growth rate will enter into a nonequilibrium state, either increasing suddenly or crashing. In a second application, Casti takes military action as the behavioural output variable; here functions which do not lead to an equilibrium state represent conflict erupting into war.

Order and Disorder in the Modern World

Human actors logically seek to control the undesirable. They try to anticipate future system states, evaluate them, analyse their presumable causes, and set about changing the antecedent conditions of future negative situations. When market processes, demonstrations or mass movements appear to get out of hand, powerful corporate actors, such as central banks, police departments and governments intervene to check spontaneous processes.

It is conceivable that situations to which models of spontaneous, unintended generation of order and disorder can be applied are becoming more frequent in modern societies. The dissolution of traditional forms of group solidarity and the erosion of hierarchies might well mean that situations which are governed by the logic of markets rather than the logics of hierarchy or solidarity are increasing.

But still only a limited part of social reality is amenable to analysis by natural science models. This is basically so because human beings are capable of purposefully organizing for the pursuit of collective goals. For better or for worse, it is not only Adam Smith's invisible hand that governs society.

The capability to organize and to formulate collective goals is closely linked to the ability of learning consciously from experience and using such knowledge strategically – instead of being restricted to learn as biological populations learn, i.e. through selective survival. Thus both individuals and, more importantly, powerful corporate actors which dominate our highly organized modern societies intervene and try to control spontaneous processes if their anticipated outcome appears undesirable.

In consequence, only a few social macro evens are therefore really emergent phenomena of the kind analysed in natural science models of nonlinear dynamics. Nor can it be said that the unintended and potentially destructive macro effects are mainly caused by processes which follow the natural science paradigm of the breakdown of order.

The breakdown of social systems is therefore rarely the consequence of the unchecked dynamic of a given nonlinear system. A theory designed to explain the collapse of social systems would have to take into account the *interference* between spontaneous developments and planned action. So far, no attempt has been made to integrate available research findings into an authentic *sociological* theory of the breakdown of social systems. The fundamental condition of social system breakdown is an imbalance between disturbances and coping – and not simply the irregular fluctuation of a given state variable. ■

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Population Dynamics and the Arms Race

By David K. Campbell and Gottfried Mayer-Kress

The new thinking about chaotic behaviour has led to the notion of "life at the edge of chaos" – expressing the principle that a delicate balance of chaos and order is optimal to life and growth on this planet.

Perhaps nowhere does this balance matter more to our continued survival than in the tangle of competing forces and values that fuel the arms race. Here chaos has traditionally been associated with unpredictability. History is replete with "what ifs" that lead down or away from the road to war. But a new interpretation, growing out of fresh ideas on chaotic behaviour, sees small-scale chaos as something that rival societies might be able to live with, without resort to wholesale violence and bloodshed.

The role that chaos theory might play in our mutual security was discussed by David Campbell and Gottfried Mayer-Kress in their report to the symposium on nonlinear arms race models. The authors are mathematicians at the Center for Nonlinear Studies at the Los Alamos National Laboratory, New Mexico, and Dr. Mayer-Kress is also with the University of California at Santa Cruz, USA. – **Editor**

The mathematical model, we believe, has great potential contributions to make to efforts to enhance global stability and cooperation. On a number of issues of overwhelming international concern – like the "greenhouse" effect, ozone depletion, or nuclear winter – better understanding of the causal relationships between specific local actions and global consequences might be obtained from studies of highly sophisticated mathematical models containing many subtle and counterbalancing effects.

The situation becomes even more complex when socio-political issues are introduced – such as population dynamics, third-world debt, or combatting the AIDS epidemic. Here, in addition to the complex technical issues, one must also try to account for the vagaries of human psychology. And obviously, both technical and human issues are very bound up in efforts to pinpoint the possible flash points of the arms race.

Beyond the problem-specific technical concerns, moreover, are challenges and limitations that arise from the very nature of the systems in which these many elements are interacting. Such systems often behave in ways that seem to defy everyday experience. For instance, the inherently nonlinear nature of these systems means that they can exhibit sudden and dramatic changes in behaviour when small changes are made in the parameters describing the interactions within the system. Further, "emergent properties" can frequently arise – that is, characteristics whose existence is not at all apparent in the initial formulation of the system.

Developing "Nonlinear Intuition"

It is crucial that those responsible for making decisions on possible courses of action be aware of this second category of general constraints and characteristics that affect the applicability and reliability of the models. To achieve this awareness, it is essential to go beyond our conventional "linear" intuition and to develop an appreciation of what can – as well as what cannot – occur in complex adaptive systems. The development of what we would call appropriate "nonlinear intuition" is extremely important. It is clear that mathematical models not only tell us what is likely to occur but also *can limit our perception of what can occur.*

Our perspective is based on the considerable recent progress that has been made in understanding nonlinear dynamical systems in the natural sciences. Two of the most important concepts in these systems are (1) *bifurcations* and (2) *deterministic chaos.*

Bifurcations are sudden, sometimes dramatic, qualitative changes in the behaviour of a system in response to small changes in the control parameters. The relevance to the modeling of complex environmental issues is immediate: the primary concern of the greenhouse effect, for example, is that slight changes in the average global temperature could lead to catastrophic changes in the climate (due to nonlinear feedback mechanisms).

$$X_{n+1} = X_n + [K11(X_s - X_n) + K23(Y_n + Z_n)]\,(X_m - X_n)$$
$$Y_{n+1} = Y_n + [K22(Y_s - Y_n) + K13(X_n - Z_n)]\,(Y_m - Y_n)$$
$$Z_{n+1} = Z_n + [K33(Z_s - Z_n) + K12(X_n - Y_n)]\,(Z_m - Z_n)$$

Deterministic chaos embodies the notion that even in those systems following precisely deterministic laws, the behaviour over long times can be essentially unpredictable, as random as a coin toss. The concept of deterministic chaos has already clarified a number of previously inaccessible phenomena, including, for instance, aspects of the transition to turbulence in fluids or the essential limits to the predictability of the weather.

Malthusian Dynamics

One of the first studied, and still most important, environmental issues is that of population dynamics. Clearly the nature and rate of growth of the human population is of central interest, both to environmental planners and security strategists. So also are a number of questions concerning the population dynamics of competing species, involving, for example, the relationships between predator and prey, parasite and host or exploiter and victim.

At the end of the 18th century, in his *Essay on the Principle of Population,* Malthus formulated the conclusion that "population, when unchecked, increases in a geometrical ratio." This model, now inextricably linked to his name, describes the growth of an isolated system evolving continuously in time in a constant environment and facing no limits to growth. The idea that population growth was inexorable and eventually had to lead to crisis had profound influence on both the evolutionary theories of Darwin and the economic theories of Ricardo.

Two generations after Malthus' Essay, the Belgian social statistician Pierre-François Verhulst formulated and named the now celebrated "logistic" equation to model the effects of limits to Malthusian growth of the population. Although there are many sources of limits to population growth, a fundamental one is the "carrying capacity" of the environment: namely, that if the population increases beyond a certain size, the environment is unable to support it, and the resulting effective reproduction rate or "fecundity" becomes negative.

It can be demonstrated through modeling that population behaviour, in certain instances, depends critically on the fecundity parameter and exhibits in certain regions "bifurcations"– sudden and dramatic changes in response to small variations in the reproduction rate. As the rate is increased, the model of population growth generates periodicity and begins to exhibit "deterministic chaos."

Arms and Insecurity

Let us turn now to the question of arms race models – and, in particular, those first introduced by L.F. Richardson in the 1920s in his work on applied dynamical systems and population growth. These models exhibit the characteristic features of nonlinearity, including bifurcations from one type of behaviour to another and, for certain ranges of their parameters, deterministic chaos.

Richardson's pioneering efforts, in his work, *Arms and Insecurity*, set the stage for subsequent attempts to analyse quantitatively many questions of strategic military and economic competition between – and among – nations. We have worked with several variants of the Richardson equations – beginning with a basic two-nation model – to show how chaos may play a role in increasing the risks of conflict. The model introduces such nonlinear variables as limited economic resources and the respective fractions of the available resources which the two nations devote to armaments.

The model shows the appearance of chaotic solutions as the countries X and Y are first in a steady state, then simultaneously bifurcate to a periodic state before together entering a situation of chaos. The transition to chaos is associated with unpredictable behaviour, crisis, unstable arms races, and therefore with an increased risk for the outbreak of war. While this interpretation can be disputed, it has become clear that the possibility of chaotic behaviour has to be taken into account in arms race models.

A second interpretation of the role of chaos in arms race models is that bounded, small-scale chaos cannot be anticipated in detail (possibly because of internal political problems of one country), but the stability of the attractor itself allows for confidence in the fact that no disastrous surprises will occur. This would be expected, for example, if there exists a basic consensus within one nation about defense policy, but details of the budget are subject to internal discussion. Elsewhere in our work, we have already suggested the effect of couplings between internal opinion formation (between"hawks" and "doves") and interactions between two nations.

The appearance of this type of small scale chaos has to be distinguished from large chaotic fluctuations which would lead to configurations in which certain situations of crises could lead to unbounded arms races, war, or economic collapse.

Chaos in "Star Wars"

In a more specific arms race situation, we analysed the impact of strategic defense systems – SDI, the so-called "Star Wars" – on the superpower arms race. This model only includes three SDI elements: (a) intercontinental ballistic missiles (ICBMs); (b) anti-ICBM satellites designed to attack and destroy ICBMs from space; and (c) anti-satellite missiles or other weapons launched to destroy anti-ICBM satellites before they can destroy the ICBMs.

The results of this model indicate that for most parameter combinations, the introduction of SDI systems leads to an extension of the offensive arms race rather than a transition to a defense-dominated strategic configuration. In the case of a strongly accelerated arms build up – either offensive or defensive – we observe a loss of stability of the solutions that we interpret as a transition to unpredictable chaos.

The Complications of Allies

When we extended the Richardson equations to model an arms race among three nations, this introduced the additional complication of possible alliances. With three nations, the natural choice for an alliance is between the two weaker nations – otherwise, the two allies would have dominant superiority over the third nation, and the competition would be reduced to that between the two nations in the alliance.

The model assumes a situation in which the three nations have arms expenditures over a given period, corresponding to a typical decision period. The equations describing armament levels take into account actual spending levels, desired rates, external threats, and economic limitations. External threats are different for nations who have to defend themselves against an alliance and those who are allies. Certain parameters determine how fast each nation tries to achieve its desired armament level, while others describe the rate at which a perceived external threat is countered. We attempted to isolate those regions of the parameters where sensitive, chaotic behaviour was obtained.

While we recognize no one could take the conclusions drawn from this over-simplified model as the sole basis for any political decision, we feel that the attempt to understand and quantify various causal relationships will lead to increasingly sophisticated models. When they are produced, one will certainly need the insight gained from the techniques of modern nonlinear dynamical systems to analyse these models.■

Enlarging Knowledge: The Road to Universality

By Mitchell J. Feigenbaum

In his pioneering work on chaos, Mitchell Feigenbaum was fascinated with the technique in physics known as "renormalization," which basically involves looking at tiny pieces of the whole structure and enlarging them. To renormalize a circle, for example, one isolates progressively smaller and smaller bits of the circle's arc. What emerges is a sequence of curves, each approaching more closely a straight line. (Consider how "flat" the round earth appears to a lone human being standing on its surface.) Renormalization turns the curve into a line – the line is thus a kind of "universal attractor."

This led Feigenbaum to begin to understand the "universality" of certain features of chaos. Seemingly unrelated transitions – the boiling of liquids or the magnetization of metals – could be governed by the same rules. His discovery implied that the equations in nonlinear systems could be besides the point: when order emerged, it seemed to have forgotten what the original equation was. Dr. Feigenbaum, who is Toyota Professor at The Rockefeller University in New York City, discussed some of the implications of his work at the Tokyo symposium. – **Editor**

I find my assigned title – "The Impact of Chaos on Physics" – somewhat constricting, and so I am going to immediately broaden it. It is difficult in certain instances to disentangle what is physics and what is mathematics, and so I shall largely ignore the distinction.

This blurring is, after all, not so surprising. It was inevitable, for example, that the creators of mechanics and the calculus were one and the same people, since the science of mechanics is largely coequal to its kinematic framework, which is the calculus. If not so grand, it is equally true that the far-flung investigations pertaining to chaos are possible precisely because of their common underpinning, which is the coming to awareness of the intuitive content of certain mathematical ideas, while the delineation of these very ideas flowed from scientific thought.

Indeed if it is true that ideas of chaos can pertain, for example, to medical phenomena, then only a thoughtless and ignorant analysis would ascribe the reason to the profound sway the science of physics holds over the others. Quite to the contrary, it is the understanding of the meaning and use of certain mathematics that happens to underlie both disciplines. Nevertheless, it is also true that the ideas of chaos have enlarged the suzerainty of physics, enabling its reach to extend to various macroscopic phenomena in a manner previously not possible.

At a much less refined level, albeit an altogether pragmatic one, this blurring of boundaries does create difficulties. Specifically – and as is clear from the scope of this conference – the studies of chaos have perilously penetrated that anathema termed "interdisciplinary." It is thus not particularly surprising to discover new young investigators rejected by one department because his efforts are "too mathematical" and by another because the work fails to pass the acid test of "sufficient rigour." Should this investigator obtain fine results, both will enjoy his company. But all too often there is a failure – to put it in coarse terms – to "put one's money where one's mouth is." Perhaps, equally vulgar, because money and funding have come to matter so much, one finds the kitty again divided by entrenched, rigid camps.

So much by way of introduction.

Art and Chaos

"In the last few years, he (Feigenbaum) has begun going to museums to look at how artists handle complicated subjects, especially subjects with interesting texture, like Turner's water, painted with small swirls atop large swirls.... 'It's abundantly obvious that one doesn't know the world around us in detail,' he says. 'What artists have accomplished is realizing that there's only a small amount of stuff that's important, and then seeing what it was.'

'I truly do want to know how to describe clouds. But to say there's a piece over here with that much density, and next to it a piece with this much density – to accumulate that much detailed information, I think is wrong. It's certainly not how a human being perceives those things, and it's not how an artist perceives them.'"

From "The Riddle of Chaos" by James Gleick, *The New York Times Magazine*, 1984

Nonlinear vs. Linear

The subject matter of "chaos" is part of the study of nonlinear systems. Yet our most potent mathematical inheritance has pertained in general to what is "linear" – and hence leaves us unprepared to investigate, as it turns out, what concerns us the most about physics. Physics consists often of the development of approximate methods of calculation because, arbitrary amounts of numerical computation not withstanding, one is powerless to plumb the contents of the theories we believe in.

My personal involvement in what is called chaos issued from a strong frustration at this inability, together with an annoyance and distaste for the various approximate methods then in vogue in high energy physics. I found it vastly more satisfying to penetrate exactly one such problem – even if not the right one – than to continue in the other mode of uncertainty and approximation. Thus, for myself, chaos emerged as one (highly interesting) manifestation of nonlinearity, whereas the attempt to erect true understandings of the mathematics that appears in our theorizing is my stronger goal.

The Routes to Turbulence

Let me now mention several specific ways in which the daily investigations of physics have been modified by the advent of chaos. First of all, the subject of the onset of turbulence in fluids has been drastically altered and, in a real sense, a new subject created. In particular, there now exists the idea of "routes to turbulence" or "regimes" in which the outcome of meticulously performed experiments in all details can be predicted. The only previously offered explanations – prior to, say, ten years ago – have been ruled out as incorrect in almost all cases. Moreover, the methodology with which an experimenter gathers his data and massages it to garner its contents has been radically extended.

The study of dynamical (oscillatory) chemical reactions has been drastically altered in the last few years. Just a decade ago, the notion that such reactions could proceed chaotically was energetically denied by some of the discipline's most distinguished researchers on grounds of "thermodynamic impossibility." The presence of chaos in these reactions has now been well observed, with theory playing some role in directing investigation.

One observes chaotic behaviour in physical systems as diverse as lasers, Josephson junctions, all sorts of fluid motions, electronic devices and their control systems, and in biological systems.... At times, these behaviours are so intrinsic and informative that researchers end up asking new questions and seeking new sorts of relations between different aspects of their data. Under these circumstances, even the mere intrusion of different modalities of relationships can provoke investigations conceivably more profitable than the associations that gave rise to them in the first place.

Impact of Chaos on Knowledge

Let me now turn to a more theoretical discussion of the impact of chaos on our knowledge and approach to general questions. Take, for example, the question of linear vs. nonlinear. Linearity means that the differential response of an object to any impressed force is independent of the state of that object, and so, mathematically described by a linear equation . The deep virtue of this circumstance is that all linear objects enjoy membership in a simple geometry of a fixed character. It is our inherited potency at visualizing and deeply comprehending this geometry that has allowed our prowess over such problems.

The situation for nonlinear problems is quite otherwise. For the purpose of physical utilization, the available mathematical knowledge is so pitifully skimpy that most problems, for which the theoretical formulation proves to be nonlinear, receive no satisfactory resolution at present. Insofar as this circumstance inheres in almost all physics investigations of present topical interest, it is almost impossible for a physicist wanting to get some results not to dabble in creating new mathematics. The study of chaos is one arena in which a predisposition to succeed in a *calculation* informed by some physical intuition has resulted in physicists putting forth what is apt to be mathematically interesting.

The tradition of mathematics of say the last half-century has come often to eschew calculation, perhaps through the realization that too much of what is considered is just specific detail and baggage that surrounds a much simpler or more transparent set of relations and considerations. Once rid of the baggage, wonderful and tight inner workings can emerge.

Sometimes, however, too much "baggage" is stripped away – so that what is general can allow only the mention of special circumstances which makes detailed investigation out of consideration. My own work on period doubling represents a case in point. The equations of analytic science tend to have a certain degree of smoothness – virtually all equations of physics, chemistry, etc. are so endowed. But this proves to be baggage in topological thought. Faced with what is perceived as an arbitrary detailed special case, topological methods are powerless. For this reason, for example, no idea whatsoever of the universal quantitative character of period doubling existed within the community of mathematicians – nor could it have. This is to reiterate that delineations of disciplines are certainly to a large measure the conventional impositions of a human community in pursuit of the present.

Impact of Universality

Let me conclude with some thoughts on the impact of universality on physics. The upshot of universality is that it is altogether irrelevant what the specific details of a physical system are in order to quantitatively deduce its behaviour.

That is, the predictable results do not depend upon the equations. How then is a particular microscopic theory to be held as explanatory towards and foundational for the phenomenon? And suppose the theory is to be held but the equations are too arduous to solve, whereas a vastly simpler theory produces exactly the same results.

What then is the human illumination that flows from the deeper theory? The question is truly and verifiably posed, and so requests honest thought towards a resolution. I will simply leave it here as a conundrum that should make the reader pause to reflect on what, if anything, the usual notion of "the scientific method" means.

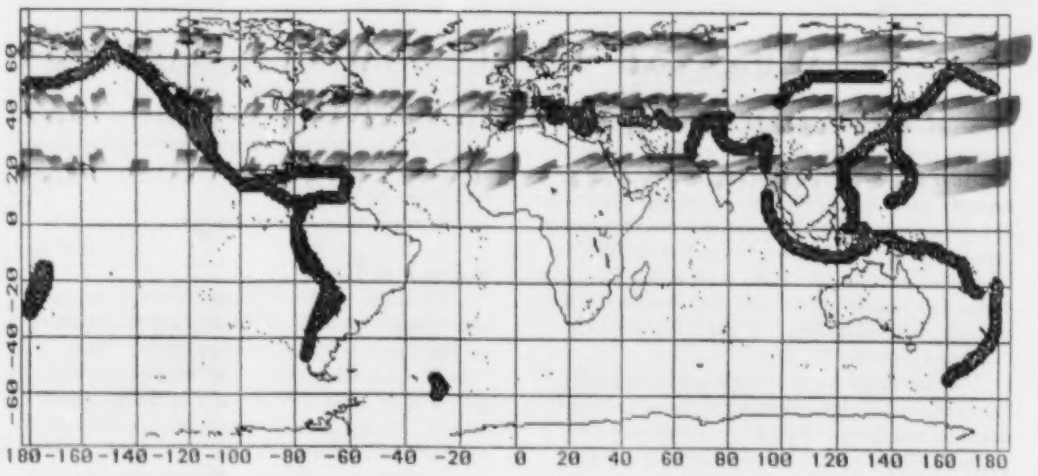
What is unquestionably true is that the theory of the onset of fluid turbulence, under certain conditions, is now well understood and measurably confirmed, despite the fact that no one yet can solve the correct equations of fluid motion. The ideas of the theory prove to be rigorous instances of "renormalization group" notions. The theory of period doubling afforded the first serious exact example of these ideas and furnished the setting for some precise understanding. Again, the ideas of renormalization in this setting proved mathematically fecund, resulting in deeper understanding of other classical results in, for example, dynamics and probability theory. ■

Improving Earthquake Prediction

By V.I. Keilis-Borok

The grinding together of blocks of varying size in the Earth's lithosphere – a crust estimated to be perhaps 80 kilometres deep – triggers some one million earthquakes a year. An estimated 15,000 people are killed annually by these tremors. The relative motion of the shifting boundary zones sets up waves, with resulting instabilities which would appear to turn the lithosphere into a nonlinear system.

Earth scientists have now begun to investigate whether the mechanisms which create this condition of nonlinearity might feature deterministic chaos. If so, it could help unlock the secrets of earthquake prediction – through study of the premonitory patterns of seismic activity. The work that he and his colleagues have done on the possible interrelationship between chaos and earthquake occurrences was discussed by V. I. Keilis-Borok of the International Institute for Earthquake Prediction Theory and Mathematical Geophysics, Russian Academy of Sciences, Moscow. – **Editor**



At the lower end of the hierarchy, interface zones separate the grains of rocks. Earthquakes make up a significant part of the movement between the blocks which form the Earth's seemingly rigid crust. The largest of these blocks are massive tectonic plates as big as continents. Friction, cohesion and relative motion between the blocks, are controlled by many diverse processes: the generation and flow of fluids; petrochemical and other transformations; fracturing, buckling, rock fatigue, and so on. All of these mechanisms turn the lithosphere into a nonlinear dissipative system which may well exhibit the characteristics of deterministic chaos.

One is stress corrosion or what is known as the "rhebinder effects" (from "rhe," the unit of dynamic fluidity), in which solids and liquids can combine through a process of cracking and interpenetration. For example, when basalt and a sulphur solution meet, the basalt is permeated by a grid of cracks, and instabilities are created.

A competing mechanism is the more conventional filtration of fluids through pores or cracks. A slip (the geological term for the distance between two points that were next to each other before movement took place) can then occur, and, with friction lessening, the movement accelerates. Instability arises as tiny slips self-accelerate, grow and merge. The very diversity of these processes suggests that we need to look for an integrated, generalized description of their actions which will help us to more clearly see the lithosphere as a nonlinear dissipative system. The fundamental equations for this system are as yet unknown. Its study, therefore, is in a particularly challenging, "pre-equation" state of phenomenology and modelling.

One of the most consequential result of our explorations to date is the understanding that the basic regularities in earthquake occurrence can be explained by the general universal properties of nonlinear systems, without evoking processes intrinsic to the lithosphere itself (essentially the principle of universality fundamental to chaos that different systems can behave identically or, from another viewpoint, there are structures in nonlinear systems that are always the same.)

What Models Show

Models first developed in theoretical physics and astrophysics were introduced in earthquake studies as early as 1967. One such model has illustrated how small displacement of elementary blocks can self-organize into strong earthquakes. The elements show prominent collective behaviour. The most stressed become self-organized in linear configurations which eventually become unstable and trigger strong earthquakes. Our experience with models suggests that we should regard the earthquake-prone lithosphere as a system remaining permanently near a critical state. Key questions can now be explored with models: the connections between the internal and the external (the "observable") parameters of the lithosphere; the minimal set of parameters required to describe the essential features of the lithosphere's short-term dynamics; the types of instabilities and transitions to chaos, and so on.

Earthquake Predictions

Earthquake prediction consists basically in a step-by-step reduction of the time and space domain where a strong earthquake might be expected. But our forecasting ability is limited by the fact that the tremors are generated by a nonlinear system of earthquake-prone faults. Our hopes for improved accuracy may lie in two opposite directions.

One arises from the notion that a strong earthquake is not an abrupt instantaneous transition within a fault system, but part of some scenario slightly extended in time. The beginning of this scenario might be used as a short-term forecast – even though it is actually a fixed part of an earthquake which has already occurred.

A second possibility is that a non-linear system may become predicable after a process of smoothing or averaging. Different degrees of averaging may yield different stages of prediction: the bigger the averaging, the larger the time-space interval in which the earthquake may be expected. This would be relevant to intermediate or long-term forecasts.

Most of the premonitory seismicity patterns analysed thus far can be interpreted as different expressions of the same phenomenon: an increase in the response to excitation. This may be expressed by the increase in the following:

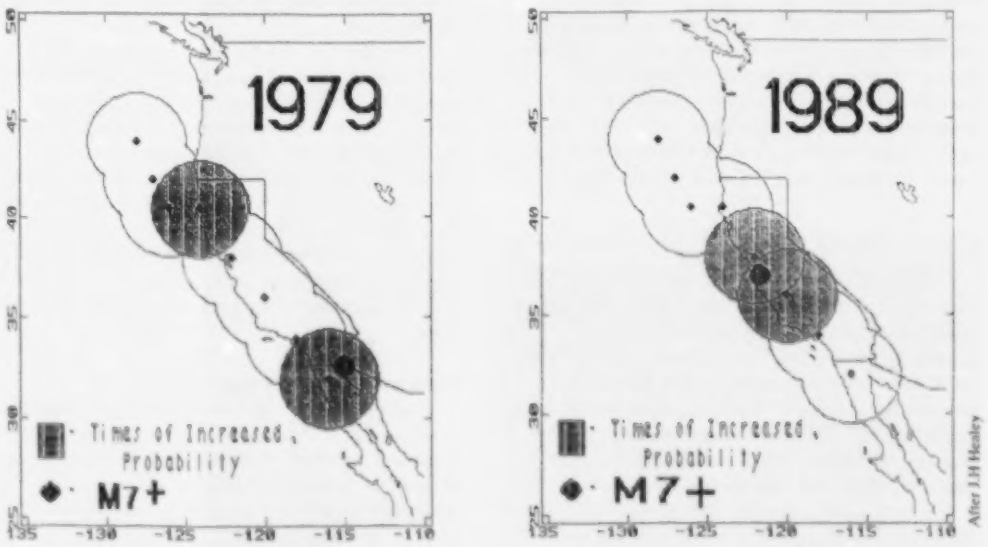
- clustering of the tremors;
- distance at which they interact;
- intensity of the earthquake flow;
- irregularities in time and space.

The approach of a strong earthquake is diagnosed when a sufficiently large number of these phenomena are observed. It must be emphasized that these phenomena are defined over large areas; they must be roughly averaged, therefore, and not treated independently.

In the present pre-equation state of the art, one must be cautious about evoking too much from the enchanting vocabulary of chaos. Still, it is clear that a number of new approaches, rooted in this field, have yielded important results for our understanding of the dynamics of the lithosphere and earthquake prediction. There have been more attempts to explain major phenomena by their universal properties, not their details. There has been recognition of the partial similarity of structure and dynamics in a wide range of energy and neotectonic environments.

Such approaches have provided an important alternative to quite opposite, and until recently, dominant tendencies. The latter had tended to focus on essential details first, in the hope of later integrating them into a general picture. The concern has been with differences, rather than similarities, and on specific mechanisms rather than their aggregation. This synthetic approach reflects the illusion that one may understand nonlinear systems by breaking them apart.

With the help of the new trends, our understanding of the lithosphere has been consolidated, the database was trimmed, and reproducible earthquake predictions emerged. This came none too soon in view of the critically growing need for earthquake prediction. The fact that this could be achieved by sheer force of new concepts even before they had been evolved into a theory illustrates how timely is the idea of this symposium. ■



Earthquake probabilities in California. With dark circles indicating an earthquake with a magnitude of M ≥ 7 on the Richter Scale. The (Loma Prieta) earthquake of October 1989, measuring M ≥ 6.9 was diagnosed in advance; the Imperial Valley diagnosis was based on data assembled after the earthquake.

How Ships Capsize

By Steven R. Bishop

In planning any structure or system, an engineer will naturally seek to minimize the possibilities of failure during all imaginable future operations – in the materials used, the design configurations adopted, appropriate use of site, and so on. But no matter how sound the long-term format, failure more often than not results from the unexpected – in a sudden wind, an unanticipated vibration, a sharp jolt from an explosion or tremour. Failure, in an engineering sense, is not a steady state – systems can fail after just a few of what engineers call forcing cycles, or periodic application of some external impulse.

*Thus elements of chaos are very much bound up in engineering dynamics. Engineers have begun to apply newly discovered aspects of chaotic dynamics to a range of their concerns, from offshore oil platforms to tensile strength of beams to flash fires in buildings. In his report to the Tokyo chaos symposium, Dr. Steven Bishop of the Dynamics Research Centre at University College, London, UK, talked about work he had done on the unexpected capsize of fishing vessels. – **Editor***

Arguably the most exciting mathematical development of recent times is the discovery of chaos and the fractal nature of many natural systems. We now know that chaos is a typical ingredient of engineering dynamics, being inherently nonlinear. This has been observed in mechanical and structural systems governed by Newton's laws of motion applicable to machine operations, vibration of structures, and the response of vehicles and other compliant objects. The associated motions are irregular; they are unpredictable and can lead to unexpected failure of what was thought to be otherwise a safe and deterministic system.

In my discussion here, I am going to preclude conservative systems, which are perhaps of greater relevance in celestial mechanics or particle physics, and consider the world typically encountered in engineering, in which some dissipation of energy tends to make the system settle down towards a steady state. However, rather than focusing on the long-term, possibly chaotic, steady state solutions which produce chaotic attractors that help identify the system, engineers have to express the integrity of a system through the transient motions from short bursts of excitation.

To understand any particular failure mechanism, the emphasis must be placed on transient dynamics – failure is a transient phenomenon and not a steady state. Typically, systems can fail after just a few forcing cycles. To illustrate this property, we consider the question of ship capsize.

Why Ships Capsized

In Canada during the 1970s it was reported that several fishing vessels capsized in sea conditions which were not thought to be particularly hazardous. To account for these unusual events, the Canadian Government commissioned a report on the nature of the stability of the common design models of fishing vessels used at the time. The overall aim of this report was to investigate whether computer programmes could be used to formulate changes to the existing requisitions for determining the necessary stability characteristics of these small vessels. The particular computer programmes used for the purpose of the study calculated the nonlinear time history in the six degrees of freedom (position and velocity coordinates for three usual dimensions), taking into account added mass, wave forces, etc.

In the report three known dangerous capsize mechanisms were considered:
1. resonant rolling;
2. subharmonic rolling;
3. stability loss at wave crest.

It is the third of these mechanisms that provides the most surprising capsize, giving little or no information of any incipient failure of the system. The hull profile plays an important role on the stability in certain positions of the vessel relative to the wave profile and it was this point that was investigated more closely.

When Capsize Happened

The conditions under which the vessels capsized suddenly without prior warning were:
1. a following sea;
2. the vessel travelling at roughly the same speed as the forward velocity o f the waves;
3. a wave length about equal to the ship's length;
4. a steep wave.

For simulation purposes, the time history was started with the vessel on the crest of a wave with the save speed slightly greater than the vessel's forward velocity. The speeds were such that after a time of about 100 seconds the ship was once again on a save crest.

Initially, a transient motion occurs before the vessel settles down to an equilibrium state. Then as the wave crest approaches amidships at about 94 seconds (see illustration), the vessel begins to roll to starboard and suddenly capsizes, turning upside down when approximately 104 seconds have elapsed.

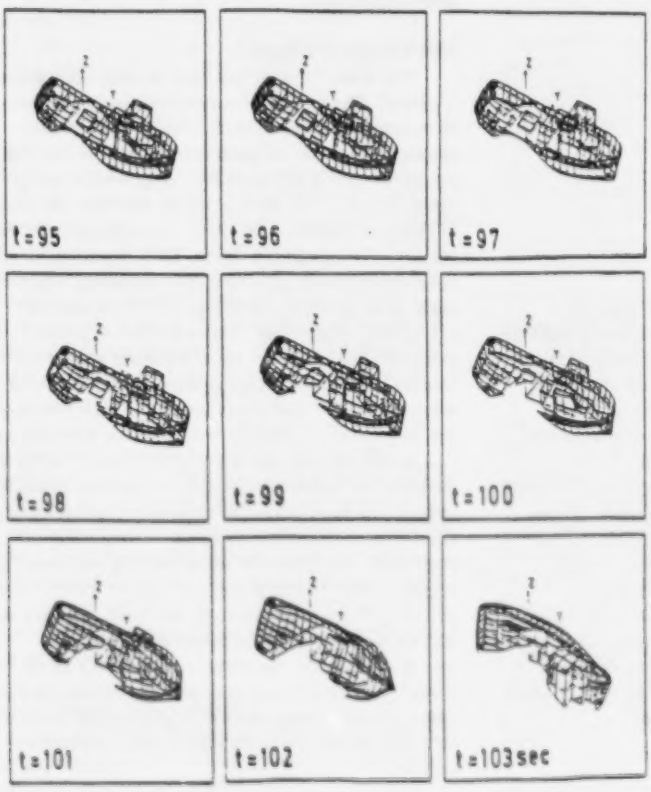
With the hull balanced on the wave crest, and the stern emerging from the wave surface, both of the vessels studied in the report experienced a loss of waterplane inertia and hence stability resulting in capsize. The use of bilge keels to increase the damping had little effect on the capsize under these conditions – with a wave crest amidships. It is possible that it is this mechanism of failure that caused the capsize, since there was no evidence that any resonant or subharmonic rolling motion occurred prior to failure.

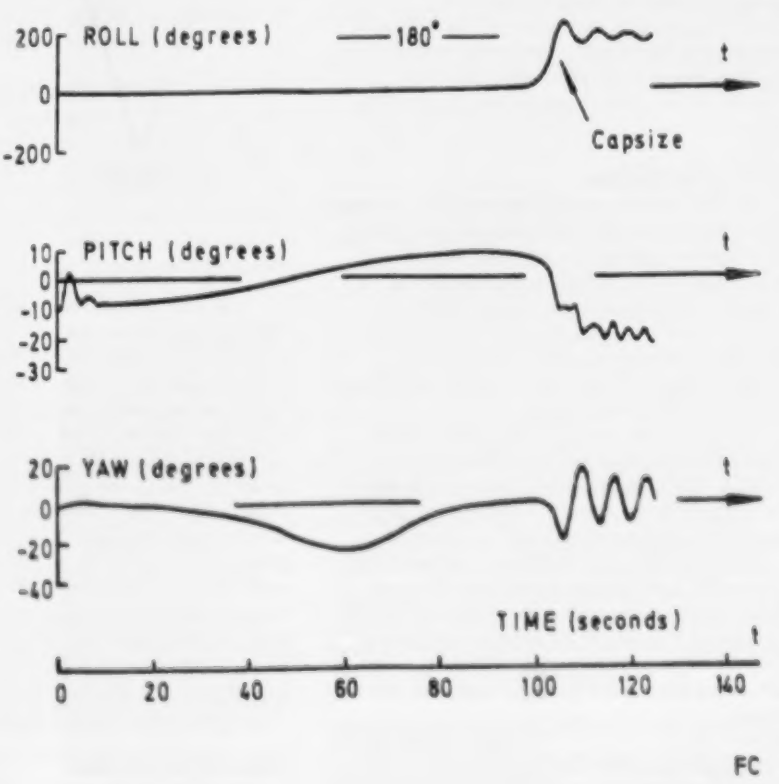
Prediction of Failure

The prediction of capsize, as we have seen, is associated with the study of transient dynamics, and not with chaotic attractors. However, the theory of chaos and methods developed for the study can be used to provide a framework for our understanding of engineering dynamical systems.

Why a Ship Capsizes

The illustrations above show the sequence as a wave overtakes a ship, causing it to roll to starboard and eventually capsize. The time between wave crests is 100 seconds. Sequence begins at 95 seconds into the roll of the wave when the crest has reached amidships. With each succeeding second, the wave upsets the ship's equilibrium further until, at about 101 seconds, the vessel reacts to the weight of the wave crest, rolls heavily to starboard, and capsizes. ■





The figures above are taken from a time sequence showing a ship capsizing in a steep following sea, with the vessel travelling roughly the same speed as the forward velocity of the waves. Less than 10 seconds elapse from the time the wave crest approaches amidships (t=95 sec.) until the ship is turning upside down (t=103 sec.).

Waking from Dreams of Determinism

By Hao Bai-lin

In the latter half of the 19th century, physicists had come tantalizingly close to the discovery of chaos, in its modern scientific sense. Poincaré approached it in his work on the stability of the Solar system. So too did Claude-Louis-Mari Navier and George Stokes in their research on turbulence. But Newtonian determinism still dominated the thinking of physicists. And interest in quantum mechanics and Einstein's theory of relativity tended to absorb the efforts of most physicists until recent decades. Theories of dynamical systems were left largely to the mathematicians. In his paper for the Tokyo symposium on chaos, Dr. Hao Bai-lin of the Institute of Theoretical Physics, Academia Sinica in Beijing, discussed some of the historical roots of the interplay between the two disciplines in the development of chaos theory. – **Editor**

In order to describe one and the same Nature, there have been two systems of description in the physical sciences: deterministic and probabilistic. Celestial mechanics, based on Newton's laws and the universal law of gravitation, had served as the touchstone for determinism. The deterministic view culminated on the night of 23 September 1846, when the German astronomer J.G. Gale discovered the planet Neptune at the first attempt, according to the theoretical prediction of the French astronomer U.-J.-J. Le Virrier. This was a triumph for the celestial determinism espoused a half-century earlier by the astronomer Pierre Laplace: physics seemed to be really able to predict the future of the universe, provided the present status was known with sufficient precision.

However, in approximately the same period, the development of thermal engines had stimulated the study of the property of gases and fluids. Here people were using macroscopic notions like pressure, temperature and volume and looking for the empirical relations between them. The laws of thermodynamics, based on an enormous collection of empirical facts, were at least as good as the laws of mechanics. The attempt to justify them by considering the elementary processes taking place in macroscopic bodies had led to the establishment of statistical mechanics.

A key point in statistical mechanics was the introduction of probabilistic elements. Traditionally, physicists preferred the deterministic to the probabilistic approach; they employed the latter only reluctantly and unwillingly. They were forced to do so when there was no better alternative, due to the complexity of the problem and the lack of knowledge or data.

Inheriting Old Tradition

The 20th century inherited this tradition. Although quantum mechanics challenged classical determinism, the probabilistic concept has only been invoked in a rather pragmatic way in interpreting and relating theoretical calculation to experimental observation. The basic equation of quantum mechanics remains linear and entirely deterministic. Consequently, determinism continued to thrive in physics, with Albert Einstein being the standard-bearer.

Although by the end of the last century, Henry Poincaré already recognized the random behaviour inherent in Newtonian mechanics, and many mathematicians complemented Poincaré's thesis, many physicists remained ignorant about these developments.

As late as 1981, A.S. Wightman complained that "... it is really a pedagogical scandal that after more than three quarters of a century the simple and enlightening ideas of Poincaré do not appear in elementary mechanics books written for physics students.... One might almost think ... that there is a conspiracy against Poincaré."

Awakening from a 300-Year-Old Dream

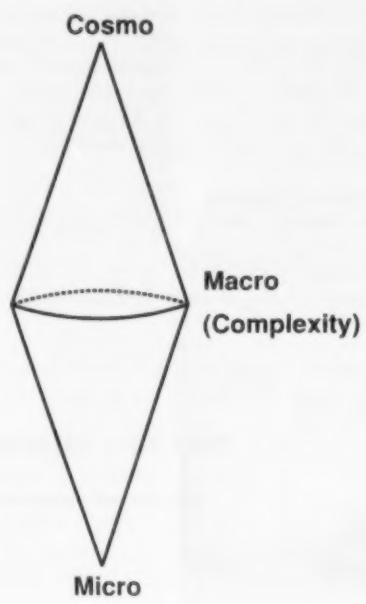
No wonder then that mathematicians like Jim Yorke and ecologists like Robert May, among others, were destined to wake the physicists from their 300-years dream of determinism. In the beginning many physicists took chaos for randomness and turned their backs. Initially, only a bunch of not-so-orthodox physicists grasped the new development with any enthusiasm.

Once convinced of the existence of chaos, however, physicists began to see it everywhere. They discover chaotic behaviour in laboratories, in computer experiments, and in observation of natural processes. The explosion of literature on chaos was to a great extent due to the efforts of physicists. (In Zhang Shu-yu's *Bibliography on Chaos*, published in 1991 by the World Scientific Publishing Company, more than 7,000 titles are listed, of which about 2,800 contain the word "chaos" or "chaotic" or "strange attractor.")

"Just Playing with Mathematics"?

When the physicists started to dig into chaos, they began to extend their vocabulary with terms which have been in use basically among mathematicians – terms like hyperbolicity, stable and unstable manifolds, homoclinic and hetereoclinic intersections, Smale's horseshoes and symbolic dynamics.

"Aha! You are just playing with mathematics," some skeptics say of their colleagues who have been deeply involved in this new game. But there is nothing strange in the situation. In the early 1920s, few physicists knew about matrix and eigenvalue problem, not to mention linear operators in Hilbert space – all notions that are taught to every physics-major nowadays.



The study of chaos emerged in the wake of breakthroughs in our understanding of phase transitions – changes from solid to liquid, from nonmagnet to magnet and so on – where the transformation can be sudden and discontinuous, the hallmarks of chaotic behaviour. Here the notion of scaling invariance and universality played a decisive role – the concept essentially that certain physical qualities have the same value, independent of any model or equation. Physicists have now mastered the skill of renormalization group techniques and are able to calculate various universal critical exponents from the linearized renormalization group equations. In this way, they have recognized structures known to mathematicians for many years – such as period-doubling cascades. A pioneering role in this area was played by our colleague at this symposium, Mitchell Feigenbaum.

Help to Other Scientists

Physicists have also been active in popularizing the new ideas in chaos theory for possible use by other branches of science. Take, for example, meteorology. Chaos has not diminished the forecasting power of science (as one might take literally from Lorenz's "butterfly effect"); it has rather dispelled any illusion about long-term forecasting powers. In fact, the study of chaos improves short-term predictions by incorporating dynamical aspects of the underlying processes. At the same time, it provides a better long-term forecasting of averaged quantities by making use of various invariant characteristics.

Speaking about the popularization of mathematical ideas, I would like to cite a little personal experience. We had found some rather complicated bifurcation and chaos "spectra" in the parameter space of several systems of ordinary differential equations. We were trying hard to understand the global structure of the parameter space and to find the systematics of periodic solutions which could be determined with confidence in numerical experiments. In so doing, we came across an old mathematical subject, namely symbolic dynamics, which has been in use in dynamical systems theory and ergodic theory since the 1920s. We soon realized that symbolic dynamics is simply what physicists call "coarse-graining," and it is a rigorous way to describe dynamics with finite precision. Therefore, it gets along with the spirit of physics very well. In fact, symbolic dynamics can be cast into a useful tool for practitioners in physical sciences.

The Problem of Quantum Chaos

In less than 20 years the "epidemic" of chaos has swept over almost all subjects of classical physics. Quantum mechanics, however, seems to have been more or less immune to the new disease.

The typical motion of a quantum system is not chaotic, but there may be some manifestation of classical chaos in finite time or frequency range – or, as some people put it, there may be a quantum signature in classical chaos. In this context, it might be appropriate to recall a comment made by Max Born, the German-British physicist who shared the 1954 Nobel Prize for his work on the mathematical basis of quantum mechanics. Born pointed out that, measured in its own natural units of time, the microscopic world, say, an atom, is much more long-living than the macroscopic world, e.g. the Solar system.

Having this comment in mind, we see the contrast: in the short-lived systems, we speak about infinite time limit and chaos, but in the eternal microworld there is no chaos. The implication of this contrast has yet to be fully understood.

New Physics in Chaos?

This leads us to the question: Is there new physics in chaos? The totality of our physics knowledge may be represented by the volume enclosed in a shuttle-shaped surface (see diagram this page). The two sharp tips of the shuttle represent the two generally recognized "frontiers" of physics: on the one hand, the study of the microworld; on the other, the exploration of the universe at cosmological scale. Both require more and more sophisticated and expensive equipment which has made these frontiers a privilege only for a selected community of scientists. However, the majority of physicists has been working at the much wider real frontier, namely the investigation of the macroworld, where we all live and where success in basic research has, in general, a quick positive feedback to the society.

In dealing with this macroworld, we are facing the problems of complexity. Indeed, the study of chaos has opened new horizons in the exploration of complexity. One of the most instructive morals we have learned from chaos consists in the understanding that seemingly complex behaviour or spatial patterns, or the combination of both, may turn out to be the results of repeated application of simple elementary rules or actions.

Who dares say that there is no new physics in chaos? It is more tempting to say the contrary, namely, chaos not only brings about new physics, but also calls for a re-examination of the fundamental principles of physics. ■

"You believe in a God who plays dice, and I in complete law and order."
Albert Einstein, Letter to Max Born

The Body's Chaotic Rhythms

Some of the most intriguing chaotic actions of all may be found in the human body – in the beat of a heart, the pulse of a brain wave, and other anatomical rhythms. Two of the papers presented at the chaos symposium dealt with this area. Leon Glass of McGill University discussed the possible role of nonlinear dynamics in cardiac rhythms and what this might tell us about heart disease. Arnold Mandell and Karen Selz considered the question of whether the tracings of brain waves by the electroencephalogram (EEG) may show evidence of a "strange attractor" which acts to integrate brain mechanisms. – **Editor**

The Beating Heart

By Leon Glass

Dr. Glass is with the Department of Physiology of McGill University, Montreal, Quebec, Canada.

The earliest experimental demonstrations of chaotic dynamics in physiological systems comes from the work carried out by Guevara, Shrier and myself in 1981. This work examined the effects of periodic electrical stimulation on spontaneously beating aggregates of embryonic chick heart cells. Periodic stimulation at a variety of different frequencies and amplitudes produced different sorts of both periodic and aperiodic dynamics.

As the frequency of the stimulus was increased, the dynamics changed from a direct 1:1 rhythm (in which for each stimulus there was a corresponding contraction or action) to other less regular rhythms. The 1:1 rhythm obtained over a broad range of stimulations. Over narrower ranges, one found rhythms such as 2:1 and 1:2. In yet other region, the rhythms were even more complex, with variations at times between four different values before returning to an original value.

If the phase of each stimulus is plotted as a function of the phase of the preceding stimulus, then one obtains a curve similar to the quadratic functions studied in detail by Feigenbaum and others. (For example, in explorations of the boundary regions between order and chaos where, essentially, one simple calculation was repeated endlessly as a feedback loop.) The appearance of a parabolic curve as well as the cascading period doubling bifurcations give evidence that this behaviour represents deterministic chaos.

Many medical conditions are associated with changes in rhythms and complex rhythms that are different from normal. Abnormalities in heart rhythms, endocrine cycles or sleep patterns are well documented in medical literature. In 1977, Mackey and I proposed that many diseases characterized by abnormal rhythmicity could be associated with qualitative dynamics displayed in mathematical models of physiological systems. We suggested that the terms "dynamical disease" is appropriate to describe medical disorders associated with abnormalities in the control systems generating and regulating the physiological rhythms. We proposed that mathematics might offer novel possibilities for the classification of disease and insight into novel therapy.

Survivorship Curves

The standard interpretation of survivorship curves is that death occurs by some random process. In 1980, however, Mackey and Lasota suggested a second interpretation of the survivorship curve, proposing that it could be associated with a deterministic chaotic process. A decade later, Mackey and Milton imagined life to be associated with a complex trajectory in some multidisciplinary space . The black regions are those that will lead to death; the deterministically chaotic life trajectory will visit such regions only very infrequently. This conceptual picture may be invoked to give qualitative agreement with clinically observed survivorship curves for patients suffering from a variety of disorder. There is good evidence that nonlinear dynamics may provide novel methods to classify post-heart attack patients who experience abnormal rhythms. In order to accurately interpret survivorship curves here, it will be necessary to have a much finer classification of the subgroups than is currently available using traditional electrocardiographic means.

The normal heart rate displays considerable fluctuation, but the source of this and the mechanisms that control it are still not well understood. Different factors regulate heart rate fluctuation, including neural systems, hormonal systems and various feedback control. In a review article from our group at McGill in 1983, we discussed the possibility that normal physiological fluctuations are chaotic. We concluded that since both neurophysiological and hormonal control systems have inherent time delays, chaotic behaviour would be expected to occur in both systems.

These and other studies make it clear that one area in which nonlinear dynamics and chaos may have an impact in medicine is in cardiology. If successful, the active research now under way will yield finer diagnoses of cardiac disease than are currently available. In order to carry out such studies, it is essential that there be close collaboration between basic scientists, physicians and mathematicians. ■

Mapping the Brain

By Arnold J. Mandell and Karen A. Selz

Drs. Mandell and Selz are with the Laboratory of Experimental and Constructive Mathematics of the Departments of Mathematics and Psychology, Florida Atlantic University, Boca Raton, Florida, USA.

The work in our laboratory on a variety of neurophysiological reactions in the early 1980s suggested that the electroencephalographic (EEG) record of human and animal brains might represent a "strange attractor." The EEG is an important diagnostic tool that maps the brain's electrical impulses. These recordings posed the possibility that the natural fractal structures of an attractor might be the instrument responsible for important integrations of brain mechanisms. Could the structure of "chaos" help to explain how the brain monitors and coordinates the constant movements of billions of bits of information?

The EEG "diagnosis" of a strange attractor was consistent with another hypothesis that we were pursuing at that time: Why was it that a number of disease states in biological dynamical systems manifested themselves via a loss in complexity? It seemed natural that a deterministic, information-generating mathematical object such as a strange attractor would be consistent with the healthy EEG and the use of this positive entropy supply by the brain for information-bearing formation.

Is the Chaotic Mind Creative?

Since that time, it has been shown that petit mal epilepsy and a case of hereditary cerebral degeneration with dementia were associated with the expected decrease in complexity of the EEG signal. Other studies seemed to support the notion that a complexity loss is associated with decreases in alertness and intellectual performance by the brain. On the one hand, it has been demonstrated that the deeper the sleep, the lower is the measure of complexity of the EEG. At the other end of central nervous system functions are the findings in which the complexity of the EEG is increased in some subjects during the initial phases of performing cognitive tasks that require recognition and response to novelty.

The theories promulgated by Smale, Ornstein and Mandelbrot, concerning strange attractors, fractals and the creation of structures, lead us to ask whether these new techniques could add to our understanding of the electrical impulses that trigger the brain. When we begin to move deeper into the implications of a strange attractor theory of the EEG to explain neurobiological mechanisms yet another theory of its dynamics emerges which may be impossible to disentangle from the strange attractor hypothesis

The arguments concerning an appropriate theoretical model for the EEG stem from around those involved with the original challenge, by Ruelle and Takens, to the Landau-Hopf theory of the onset of hydrodynamic turbulence. In this previously widely-accepted concept, turbulence arose essentially from the superimposition of three or four periodic motions – or "wobbles" – which become larger and larger. These motions or frequencies take the form of a torus, the surface generated by a circle rotated about an axis in a plane that does not intersect the circle.

The Ruelle-Takens argument was that three such superimposed frequencies actually combined to create a new object – which they called a "strange attractor." They further argued that as more than three frequencies (greater than 3-tori in mathematical jargon) came into play, chaotic solutions became both more frequent and more structurally stable.

Three "Symmetries" of the Brain

The implications of this are particularly interesting for the brain, which has three spatial-functional symmetries: anterior-posterior (of future and past, of imagining vs. seeing); right-left (analytic-algebraic vs. geometric); and up and down the neural axis. These symmetries oscillate, which make issues of expected behaviour vs. exceptional departures different than they would be if studied only in time. The spatial symmetries of the brain may make it more likely that this organ possesses some form of "nongeneric" – that is to say, non-typical – stability.

One of the persistent mysteries of the EEG is the source of the so-called "slow" rhythms, which range from 1 to just over 50 cycles per second (Hz). The most characteristic are: (a) under 4 cycles per second, deep sleep or coma; (b) 4-7 Hz, day dreaming and light sleep; (c) 10-12 Hz, relaxed, eyes closed, awake; (d) 18-30 Hz, very alert, aroused or anxious; and (e) 35-50 Hz, periods thought to be associated with instances of spatial coherence in perceptual tasks.

These "facts" from over 60 years of research on the human EEG have never been disputed, yet they have never been well understood. The strange attractor theory has led to serious questions concerning the usual analytic approach to spectra with multiple broad band modes. The strange attractor theorist views such spectra in another way – as the instabilities and aperiodic actions of expanding and folding dynamical systems. They are the result of the post-Landau-Hopf global bifurcation phenomenon – the scenario of Ruelle and Takens – and are *undecomposable*. Ruelle refers to these complex singularities of the power spectrum as "resonances" of the dynamical system. ■

Disorderly Creativity

"Those studying chaotic dynamics discovered that the disorderly behaviour of simple systems acted as a creative process. It generated complexity; richly organized patterns, sometime stable and sometimes unstable, sometimes finite and sometimes infinite, but always with the fascination of living things."

James Gleick, *Chaos: Making a New Science* (Viking Penguin, New York/London, 1987)

UNU UPDATE

The following is taken from the Rector's Report to the Council of the United Nations University for the year 1991. – **Editor**

• The Rector, Professor Heitor Gurgulino de Souza, was appointed by the Secretary-General of the United Nations to a second five-year term beginning in September 1992.

• The UNU Council met in December in the Toho Seimei Building for the final time. By the time of the next Council session, the University will have moved into its new permanent headquarters in Tokyo, a building designed by the noted Japanese architect Kenzo Tange and constructed by the Government of Japan.

• Dr. Nevin Scrimshaw, the Director of the UNU programme on Food and Nutrition for Human and Social Development, received the World Food Prize for his outstanding contribution in the area of nutrition. He was also awarded the Alan Shawn Feinstein Award of the World Hunger Program of Brown University (USA) for research and education.

• The UNU is making significant contributions to the 1992 United Nations Conference on Environment and Development (UNCED). The report of the UNU World Institute for Development Economics Research (UNU/WIDER) on *A Global Environmental Compact for Sustainable Development* was quoted extensively and adopted by the preparatory Committee for UNCED. The UNU's programme on Mountain Ecology and Sustainable Development will take the lead in formulating the mountain agenda for the UNCED, with substantial support from the Swiss Development Cooperation. The programme on Human Dimensions of Global Change is also making an input through reports and recommendations stemming from the UNU Conference on International Governance and Global Environmental Change and the project on International Law and Institutions. The programme on Sustainable Development in the Humid Tropics will also add to the UNCED deliberations.

• An international symposium, jointly organized with the International Peace Research Institute, at the UNU headquarters in Tokyo in September discussed the future prospects of the United Nations Peace-keeping Operations and emphasized the role of civilians.

• A major meeting in November in Rome, supported by the Italian Government, presented the results of the Project on Household, Gender and Age. Analysing the the impact of macro-events on women and the consequent changes in the household, the project conducted case studies in Argentina, Brazil, Chile, China, Colombia, Kenya and Sri Lanka.

• Steps toward starting up operation of the Institute for Natural Resources in Africa (UNU/INRA) continued in 1991 with financial support of US$1 million from UNDP. The Government of Zambia has paid $1.2 million of its pledge of $2 million in support of INRA's mineral resources unit at the School of Mines at the University of Zambia. The Government of Ghana has made a commitment to host INRA's main centre and provide $4.75 million to the University's Endowment Fund for the Institute, and has recently paid the first tranche of US$1 million. Ghana has previously contributed US$2.5 million to the Endowment Fund, of which $250,000 was earmarked for the Institute. The Programme for Natural Resources in Africa, which is designed to lead to the full realization of INRA, continued to implement activities.

• UNU sponsored an international conference on the culture of violence in Lima, Peru in October, with the Peruvian Peace Research Association. The conference reviewed examples of routinized violence and is impact on life in Latin America, Asia and Africa.

• In Villeta, Colombia, experts instructed 22 participants from eight Andean nations at a month-long training seminar in October on conflict resolution, organized in cooperation with the Pontificia Universidad Javeriana. A similar Asian seminar was held in Diliman, Philippines, in November with the University of the Philippines Centre for Integrative and Development Studies.

• The research of the UNU World Institute for Development Economics Research (UNU/WIDER), is having a profound impact on policy-making at the domestic and global levels, particularly in the areas of the reform process in Eastern Europe, structural adjustments in developing countries, and the resource transfer implications of environmentally sensitive development in the developing countries.

• The UNU Global Environmental Forum: Monitoring and Action for the Earth (supported by the Japanese Environment Agency, the Ministry of Foreign Affairs, the Ministry of Education, Science and Culture, the Ministry of Construction, and Japan Broadcasting Corporation (NHK), and sponsored by Obayashi Corporation) held in Tokyo in July brought together internationally renowned scientists and Japanese scientists to discuss major issues and new methodologies. The event received wide media attention in Japan; a video documentary has been produced by the NHK network .

• An international workshop on mass production of photovoltaics was held in collaboration with UNCSTD in São Paulo, Brazil in September. The strategy for mass production of solar cells, in both industrialized and developing countries, that was developed will be considered at the 1992 UN Conference on the Environment and Development

• The Japanese City of Minamata, which has become a byword for a toxic environment, was the site of a UNU conference in November that examined problems of heavy metal pollution and related health effects. The meeting was jointly sponsored by the City of Minamata and Kumamoto Prefectural Government.

• A regional training workshop on scientific and technological information management was held in New Delhi in October as part of the programme on science and technology for development. The session was held in cooperation with UNESCO/ROSTSEA, the Science and Technology Policy Asian Network, and the Indian National Institute of Science, Technology and Development Studies.

• The first meeting of the plant biotechnology network was held from 11 November–6 December at the Universidad de Rosario, Argentina, as part of the work of the programme for Biotechnology in Latin America and the Caribbean (UNU/BIOLAC). A course was also organized on control of Pertussis vaccines at the Universidad Nacional de la Plata in Argentina.

• The Government of Ireland pledged 240,000 Irish punts to support a three-year micro-informatics project by the UNU to develop institutional capacities of universities in developing countries. The training will be provided at Irish universities with follow-up workshops in the home countries of various trainees.

• An international conference on information technology for social purposes was held jointly by UNU and the Universidad de las Palmas in the Canary Islands in November. The University also collaborated with the International Centre for Theoretical Physics and the University of Zimbabwe in the Fifth International Workshop on the use of microcomputers in science and mathematics education in Africa in January 1992.

• The Ph.D. internship programme began in the last quarter of 1991 at the Institute for New Technologies (UNU/INTECH) in Maastricht, the Netherlands. In addition to its research activities, the initial work of UNU/INTECH is placing great emphasis on the programme. The effort brings three to four Ph.D. candidates from other universities to the Maastricht centre for a period of about three months.

• The UNU sponsored a symposium on rapid assessment procedures (RAP) and played a significant role in discussions on iron deficiency and nutrition composition of foods at the Sixth Asian Congress of Nutrition in Kuala Lumpur, Malaysia, in September.

• The initial phase of the new UNU project jointly with UNRISD on the socio-economic and political consequences of the international drug trade has been completed. This is a literature survey accompanied by a bibliography; in a second phase, country reports are being prepared on Bolivia, Colombia, Laos, Myanmar, Pakistan, Mexico, Peru, Thailand, and the United States (in Kentucky). ■

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Consulting Editor: John M. Fenton (Connecticut, USA)
Administration, Production and Distribution: Sumiko Sudo **ISSN 0259-4285**